Please use our materials!

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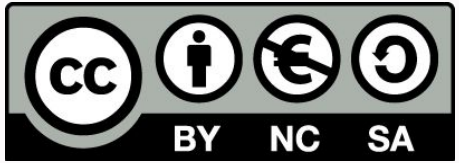
We also humbly request that you email sarah.adams@olin.edu if you use these materials, as we are tracking their impact and how far they travel!



How To Win Games Every Time

HELLO!

Junwon

(he/him)

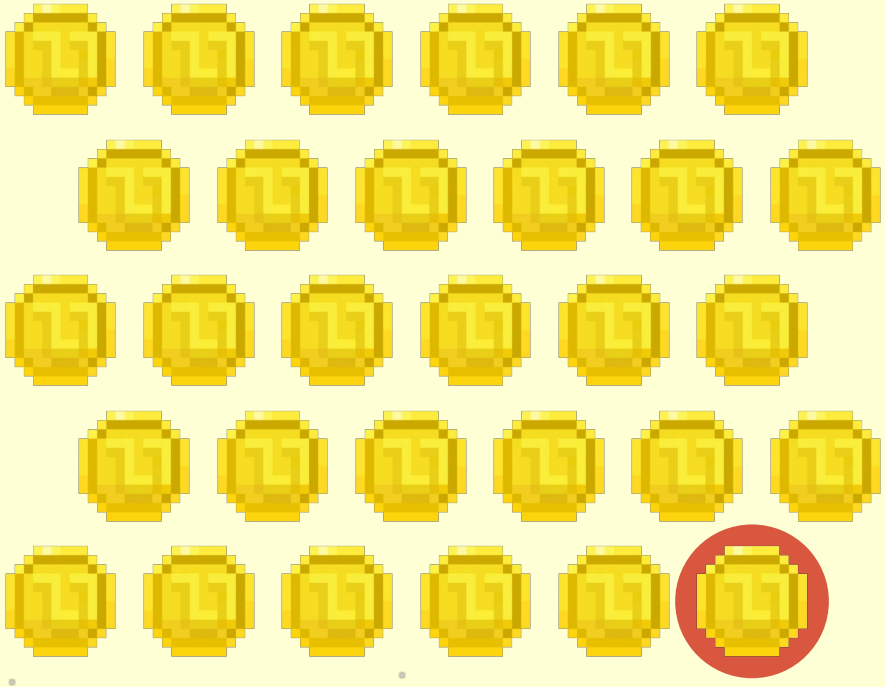
Reid

(they/them)

Ashley

(she/her)

Coin Game! Let's play again!



= Teacher = Students

Let's take turns X-ing out coins!

There are 30 coins on the slide.

You can X out up to 4 coins each turn

The player to take the last coin is the **loser.**

THE GAME WAS RIGGED!

Player 1 can win every time if they know the strategy!

So... What is the strategy?

Let’s back up and see how that strategy played out in the game we just played

Coin Game! Let's play again!

Let's take turns X-ing out coins!

There are 30 coins on the slide.

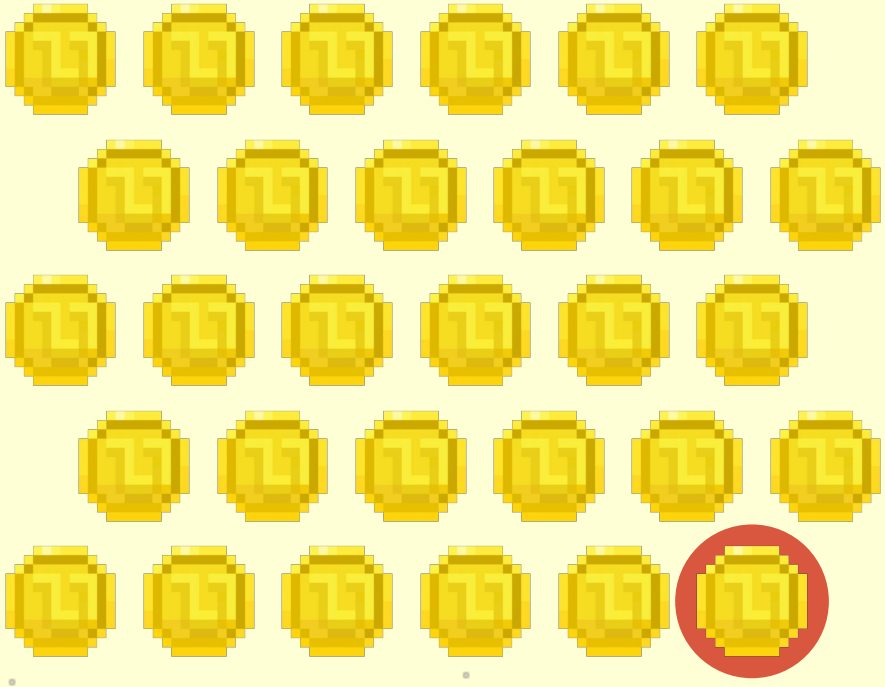
You can X out up to 4 coins each turn

The player to take the last coin is the **loser.**

The Strategy for Player 1

Step 1: To start, **take 4 coins**
Step 2: Note how many coins player 2 takes on their turn
Step 3: **Take 5 *minus* the number of coins player 2 just took**
Step 4: Repeat until you win!

Your turn to go first!



= Students

= Teacher

There are 30 coins on the slide.

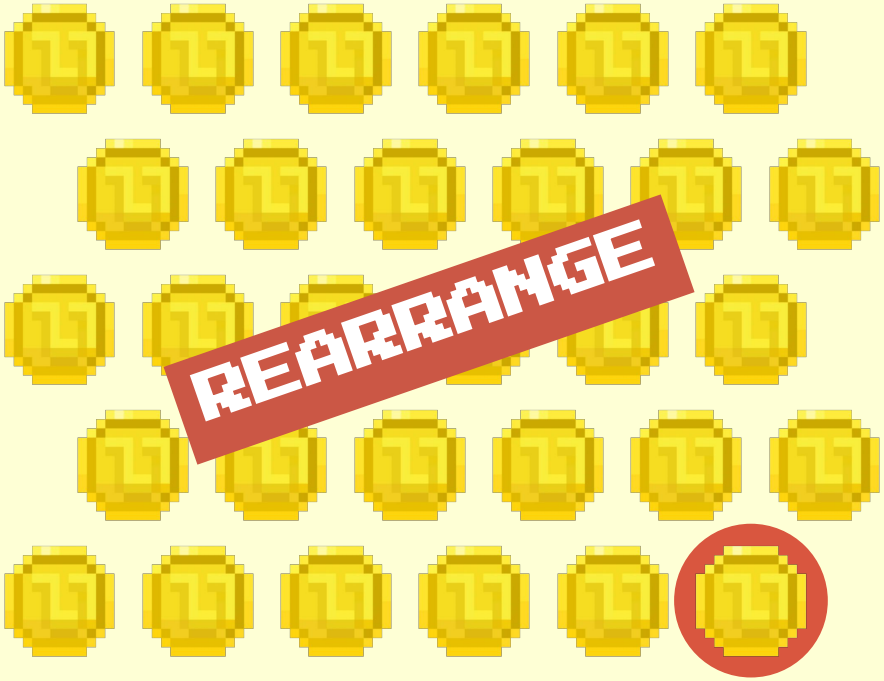
You can X out up to 4 coins each turn

The player to take the last coin is the loser.

Remember, to win:

- **Take 4 coins the first time**
- **Take 5 minus the number Junwon took after that**

Why does it work?



30 coins

Students go first as Player 1

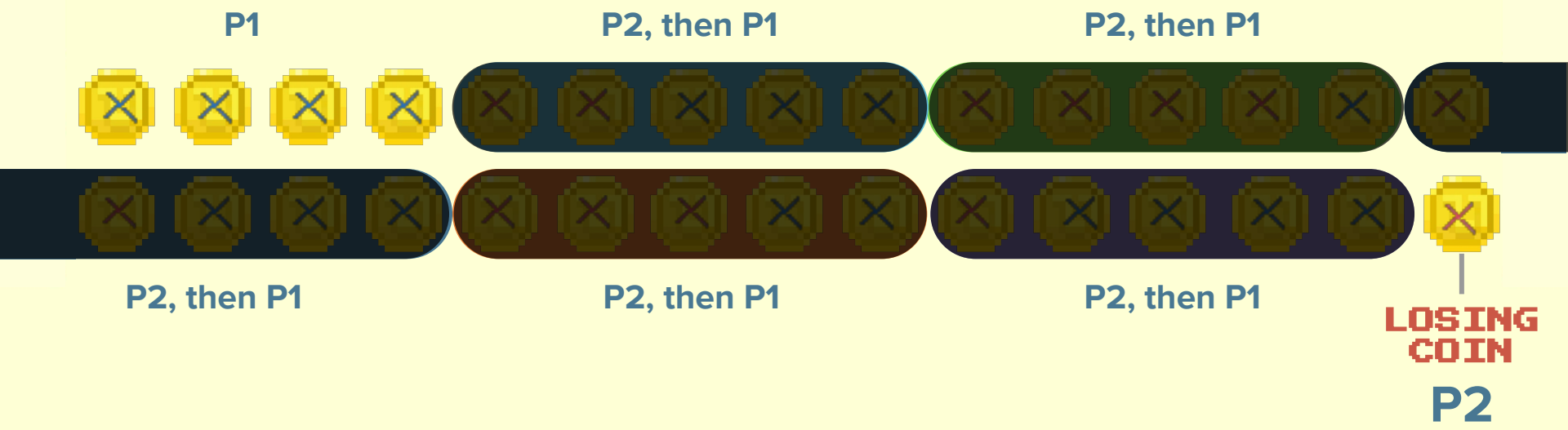
LOSING COIN

Player 2

= **Students** (player 1)

= **Teacher** (player 2)

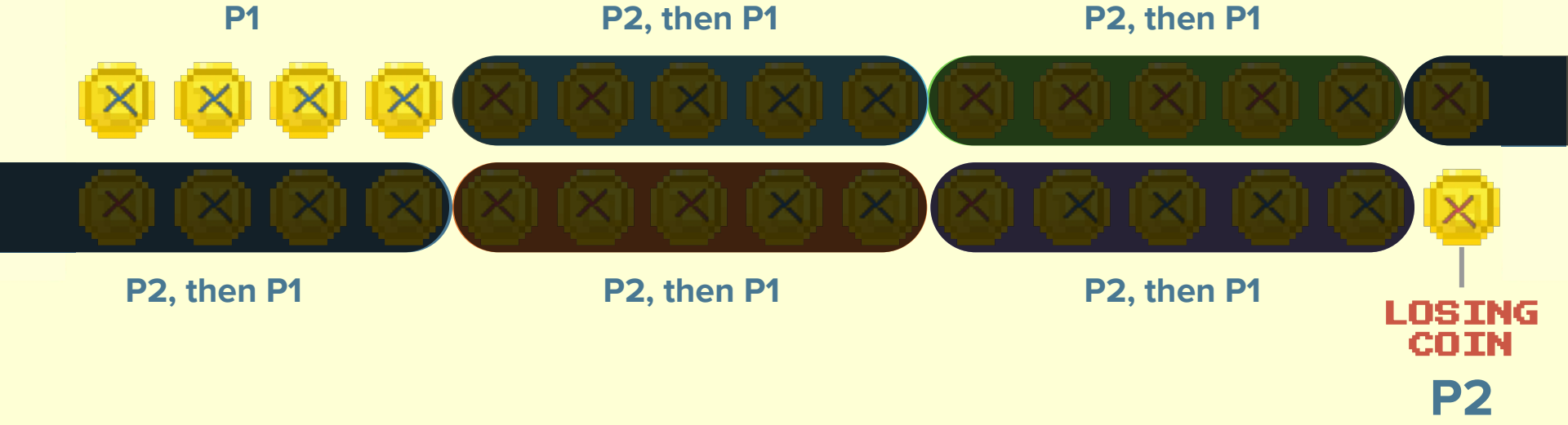
Why does the strategy work?



= **Students** (player 1)

= **Teacher** (player 2)

Let's start from the end



= **Students** (player 1)

= **Teacher** (player 2)

Why subtract from 5?

1 (the minimum # a player can take) **+ 4** (the maximum # a player can take) = **5 coins**

Teacher takes **1 coin**

Adds to 5 ✓

Teacher takes **2 coins**

Adds to 5 ✓

Teacher takes **3 coins**

Adds to 5 ✓

Teacher takes **4 coins**

Adds to 5 ✓

5 is the ONLY number that works

We can't always make a round add to 6 coins:

This only adds to **5 coins**

Teacher takes **1 coin**

Students can only take up to **4 coins**

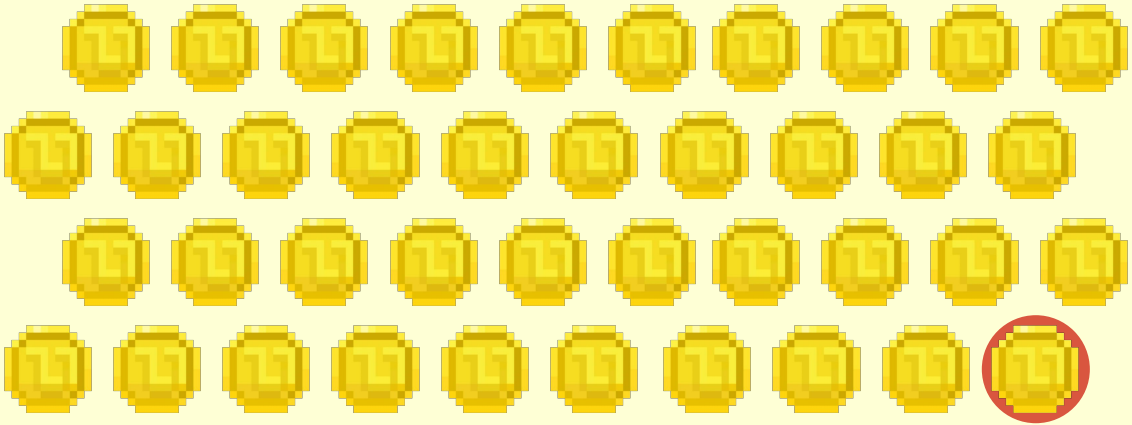
We can't always make a round add to 4:

This adds to **5 coins**

Teacher takes **4 coin**

Students have to take at least **1 coin**

Let's come up with strategy for another coin game!



How does player 1 win this?

Let's take turns X-ing out coins!

There are **40** coins on the slide.

You can X out up to **6** coins each turn

The player to take the last coin is the **loser.**

Player 1 goes first

Why is 7 the "magic number" now?

1 (the minimum # a player can take) + **6** (the maximum # a player can take) = **7 coins**

Teacher takes **1 coin**		We take **6**
Teacher takes **2 coins**		We take **5**
Teacher takes **3 coins**		We take **4**
Teacher takes **4 coins**		We take **3**
Teacher takes **5 coins**		We take **2**
Teacher takes **6 coins**		We take **1**

7 is the only number that works now

We **can't** always make p1's turn and p2's turn **add to 8** coins:

This only adds to **7 coins**

Teacher takes **1 coin**

Students can only take up to **6 coins**

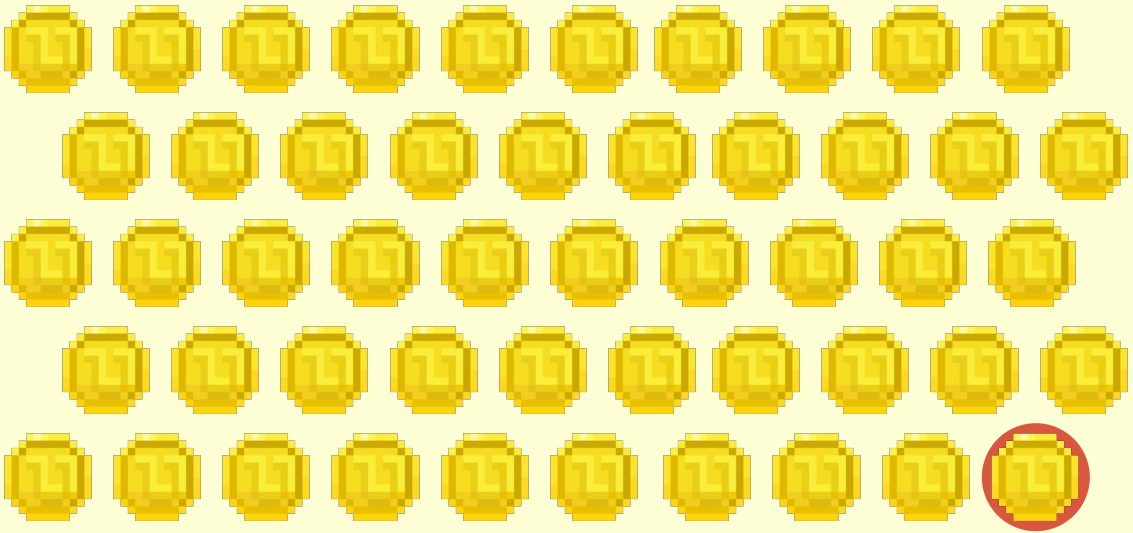
We **can't** always make p1's turn and p2's turn **add to 6** coins:

This adds to **7 coins** :(

Teacher takes all **6 coin**

Students have to take at least **1 coin**

But Does This Work Every Time?



Let's take turns X-ing out coins!

There are **50** coins on the slide.

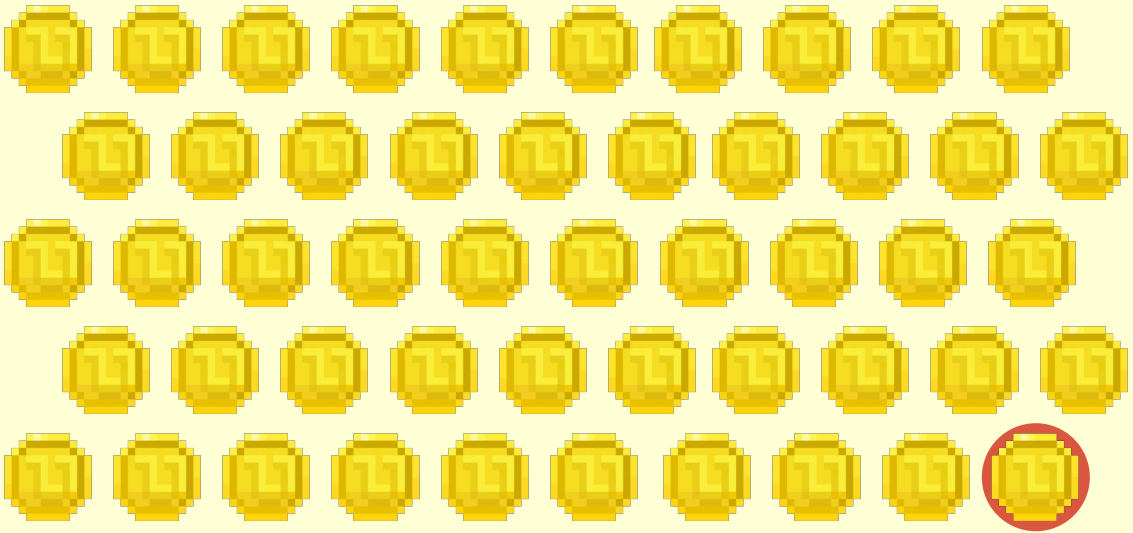
You can X out up to **6** coins each turn

The player to take the last coin is the **loser.**

Player 1 goes first

How does player 1 win this?
... Or can they?

But Does This Work Every Time?



Let's take turns X-ing out coins!

There are **50** coins on the slide.

You can X out up to **6** coins each turn

The player to take the last coin is the **loser.**

Player 1 goes first

How does player 1 win this?
... Or can they?

IMPOSSIBLE!

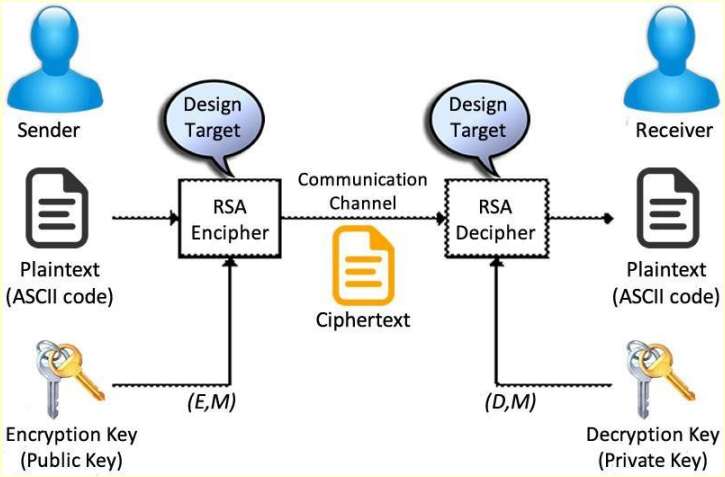
By The Way...

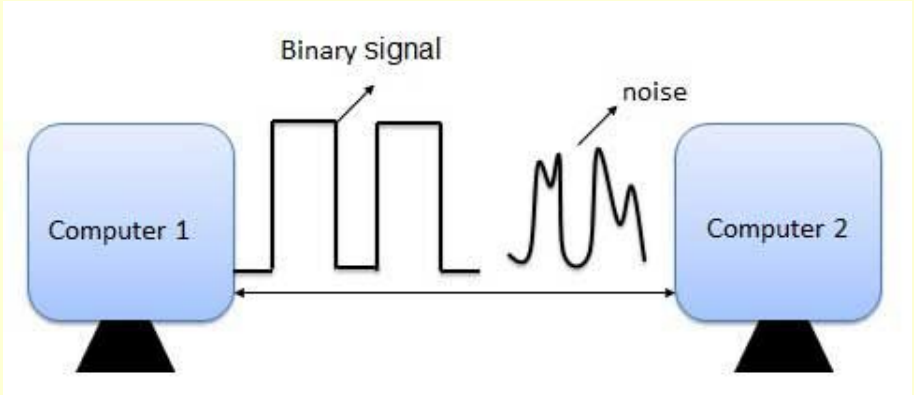
What math did we do other than adding & subtracting numbers?

MODULAR ARITHMETIC !!!

But what is Modular Arithmetics?

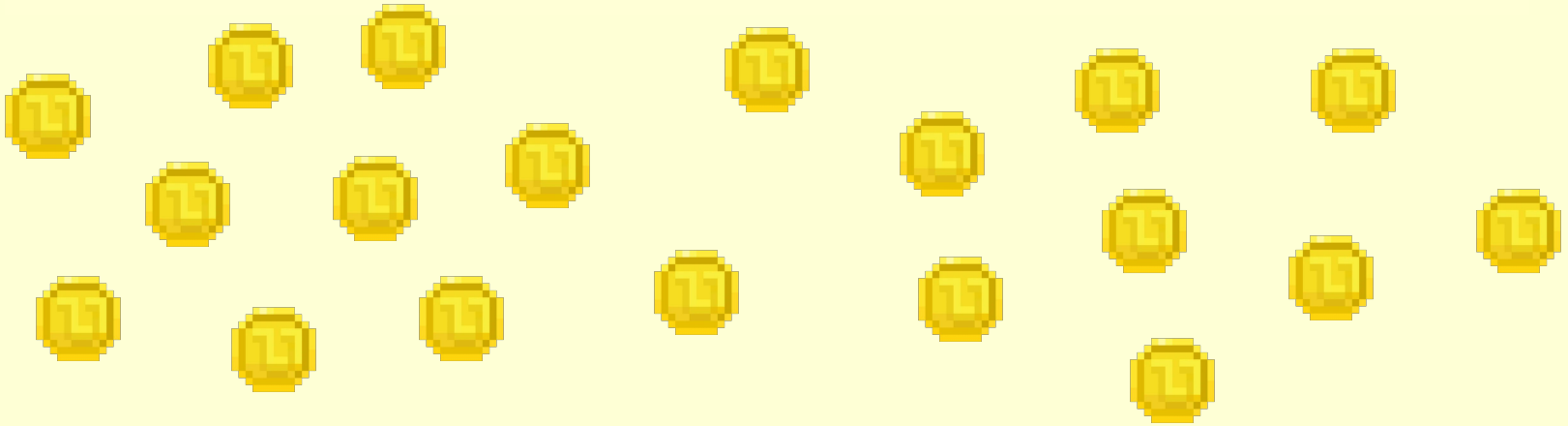
- Modular Arithmetics is arithmetics, but with remainders!
- Observe numbers with extra attention to divisibility rules and remainders
- Extensively used in cryptography, communication systems, and so on!





Where is Modular Arithmetics in Coin Game?

- The remainder is the “last coin”
- Dividing the coins up into sums that we can control
- Same strategy of taking coins no matter how many coins we have!



Create your own coin game!

Come up with your own coin game!

Things you can change

1. # of coins in a pile
2. # of coins collected each turn
3. Whoever takes the last coin is the winner
4. What if there are 3 players?
5. What if there are 2 piles of coins? (i.e. there are two piles of coins, you can only collect coins from 1 of 2 piles each turn, whoever picks last coin is the loser.)

How do I get this strategy?

Questions to consider asking yourself:

1. **What is the goal** of the game?
2. **What can I control** in this game?
3. How do I use what I can control to **guarantee victory**?

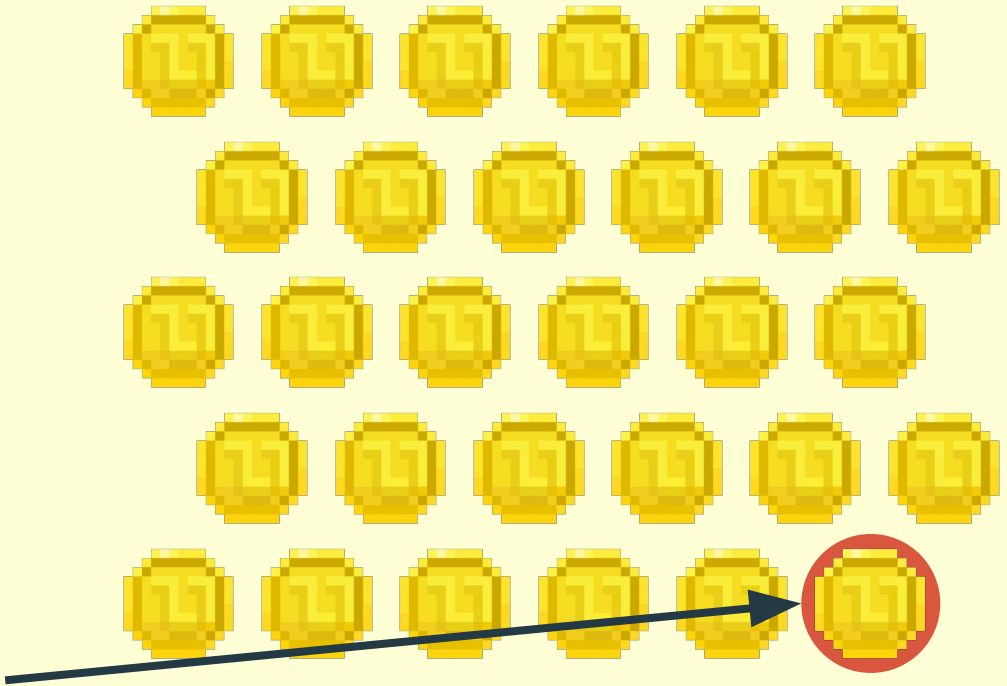
What are the answers to these questions in the coin game?

Talk in your breakout room for 5 minutes.

How do I get this strategy?

1. What is the goal?

   To **NOT** get the last coin, to leave it for player 2

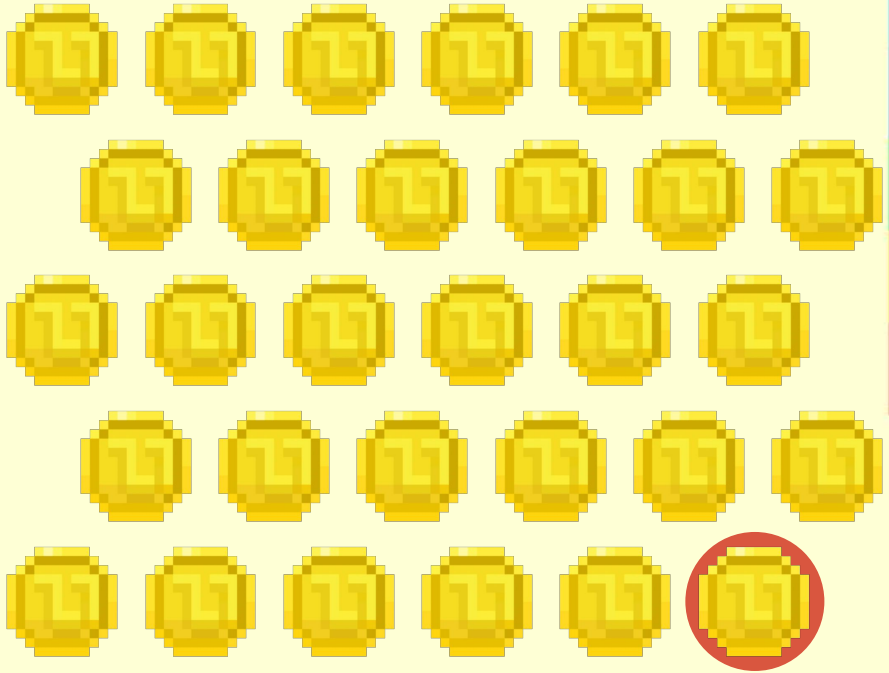


We don't want this!
We want this to be given to player 2

How do I get this strategy?

2. **What can I control** in this game?

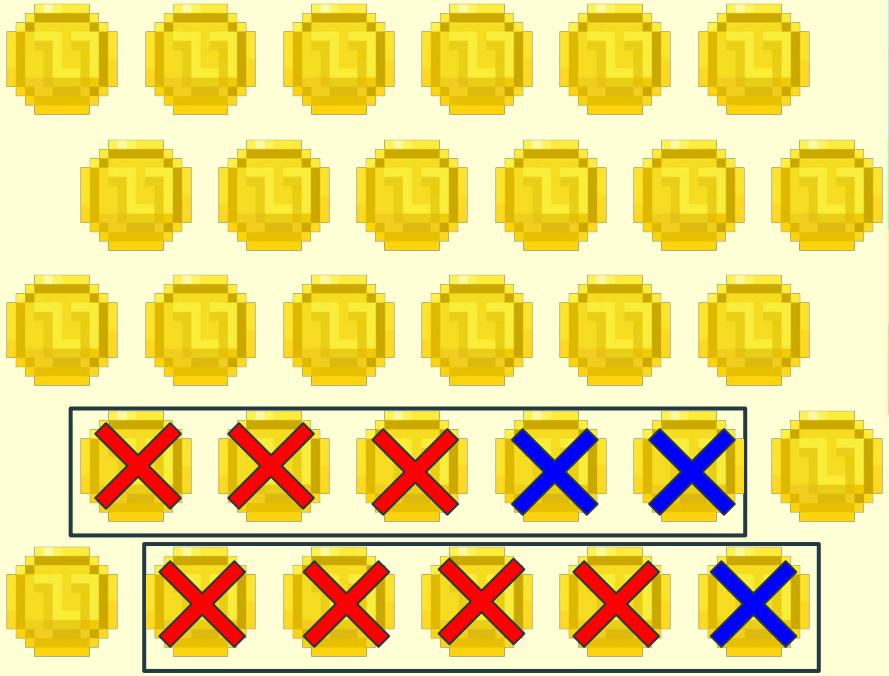
- I can choose the **number of coins I take** each turn
- I can control the **sum** of my coins + my opponents coins.
    - I can make sure the sum is **always the same number**



How do I get this strategy?

2. **What can I control** in this game?

If my opponent take their turn, I can collect coins so that the sum of coins collected from the opponent and me will be 5 (this can be guaranteed).

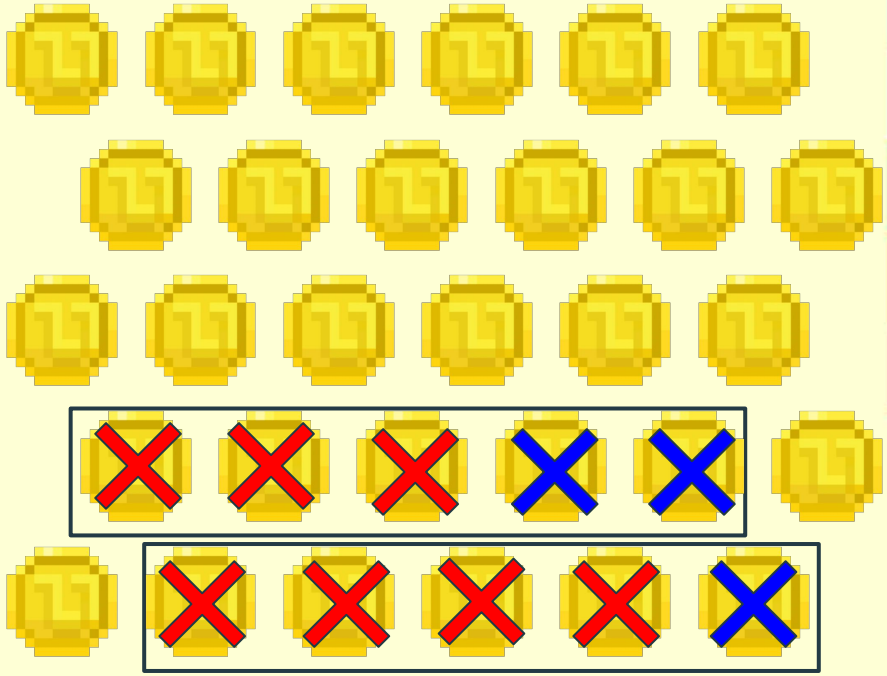


Player 1

Player 2

How do I get this strategy?

If my opponent take their turn, I can collect coins so that the sum of coins collected from the opponent and me will be 5 (this can be guaranteed).



Player 1

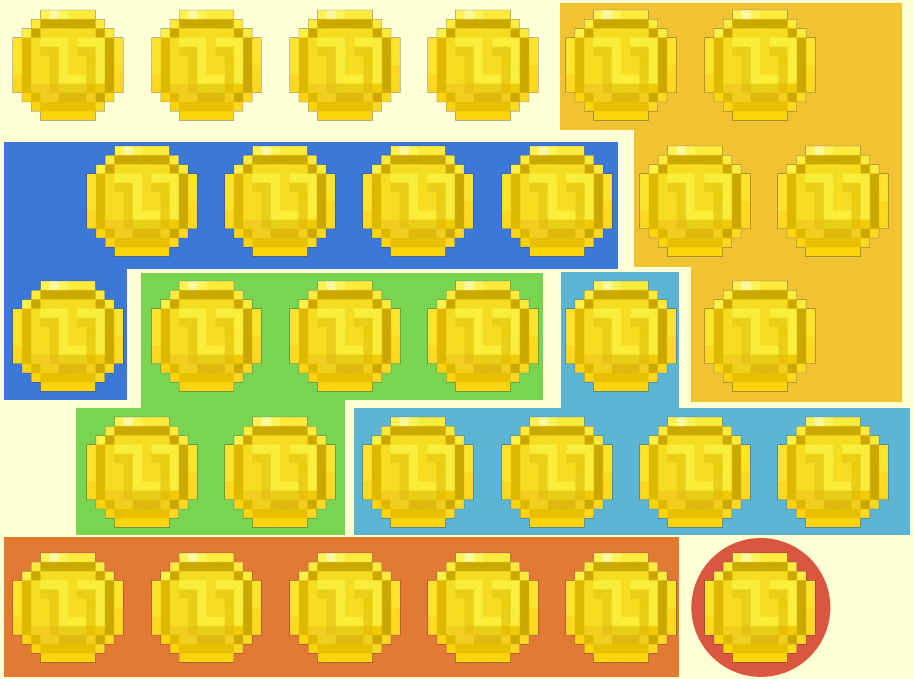
Player 2

How do I get this strategy?

3. What must I do to guarantee victory?

- We want to take up every coin **except** the last one. With 30 total coins, that means we need to take up **29** coins in order to leave one for our opponent
- We know that we can guarantee a total of **5** coins taken per turn.

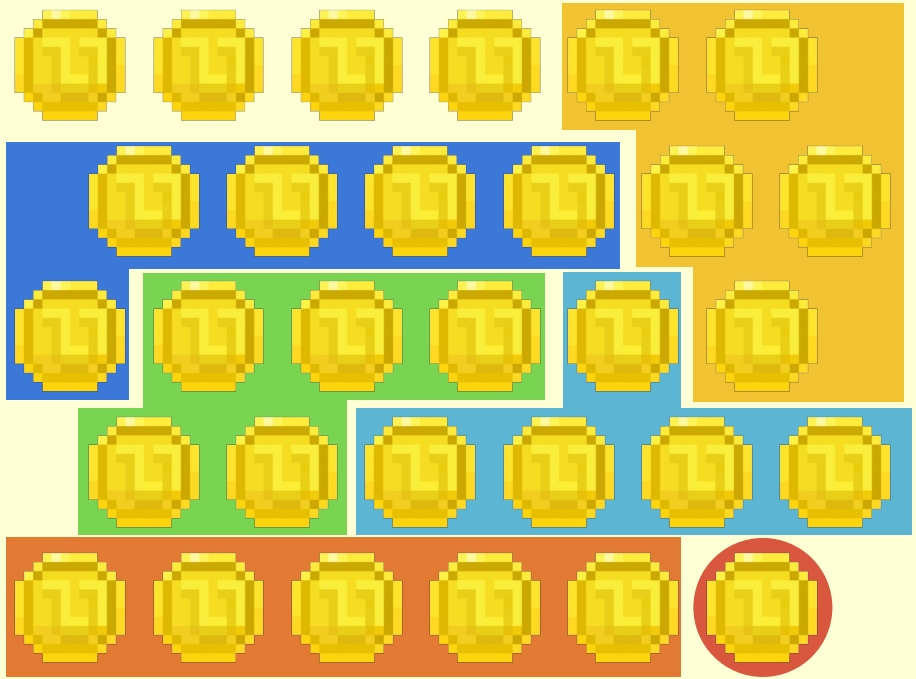
- This means we know we can *guarantee* the fate of **25** coins



How do I get this strategy?

3. What must I do to guarantee victory?

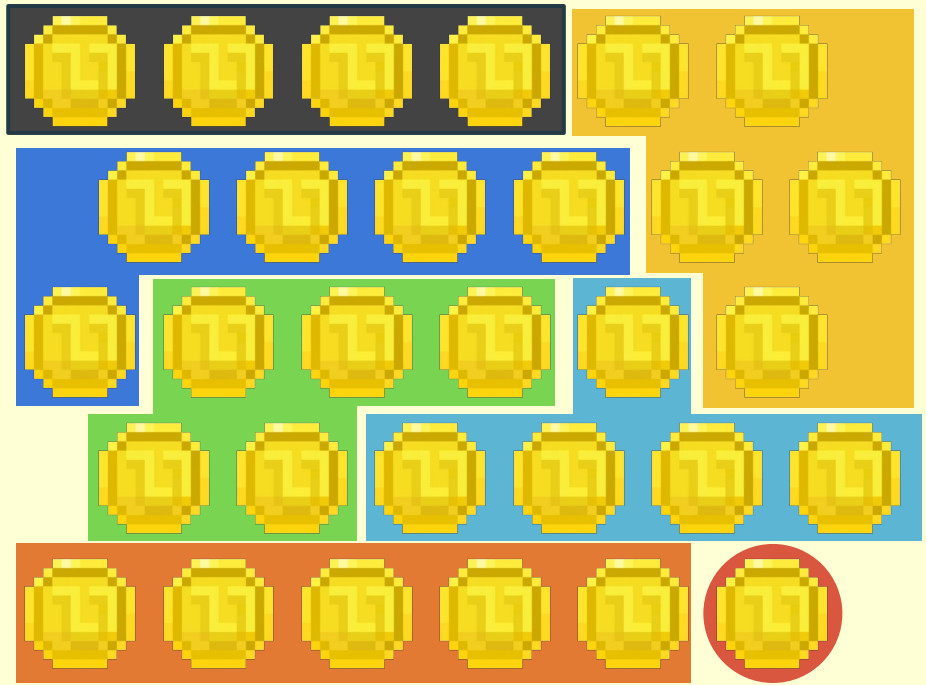
- If we can control the fact that 25 coins will be taken, how can we win the game?
- Does it matter if we go first or second?
- Is there a number of coins we can take on our first turn to **be sure** that we'll win every time?



How do I get this strategy?

3. What must I do to guarantee victory?

- By taking the **first four coins**, Player 1 can make sure they win!
- They still need to make sure each turn sums to **five** total coins



The last coin is still left for Player 2!